MAGNETOELASTIC RELATIONSHIPS FOR LARGE DIAMETER TERFENOL

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and

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I. Introduction

The origins of this position paper lie in discussions about the proper form of the compliance matrix and its inverse, the modulus matrix, appropriate for Terfenol-D. With very little additional work, we can discuss the full piezomagnetic matrix which includes not only the compliance matrix but also the piezomagnetic constants and the permeability constants. The general matrix has 45 independent constants, however these can be reduced to 5 independent constants by applying symmetry appropriate to today's high-power Terfenol-D drivers. We also briefly discuss methods of measuring these constants.

II. Table of symbols

```
d
         effective piezomagnetic constant (A/m)
         effective piezomagnetic constant (m/A)
e
         effective piezomagnetic constant (T)
g
         effective piezomagnetic constant (T-1)
h
SH
         elastic compliance, H = \text{constant } (\text{m}^2/\text{N})
_{S}B
         elastic compliance, B = \text{constant (m}^2/\text{N)}
c^H
         elastic stiffness, H = \text{constant} (N/m^2)
c^B
         elastic stiffness, B = \text{constant} (N/m^2)
Н
         magnetic field strength (A/m)
В
         magnetic flux density (T)
\mu^T
         permeability, stress = constant (H/m)
\mu^{S}
        permeability, strain = constant (H/m)
\nu^T
         reluctivity, stress = constant (m/H)
vS
        reluctivity, strain = constant (m/H)
\varepsilon, S
         strain
\tau, T
         stress (N/m<sup>2</sup>)
\boldsymbol{E}
         Young's modulus (N/m<sup>2</sup>)
```

III. General Elastomagnetic Matrix

In tensor form, the four basic linear relationships for the piezomagnetic materials become:

```
s^H \delta 	au
                                                           + d\delta H
\delta \varepsilon
\delta B
                    =
                                       d \delta \tau
                                                           + \mu^{\tau} \delta H
                                       c^H \delta \varepsilon
\delta \tau
                                                       - ебН
\delta B
                                       e \delta \varepsilon
                                                            + \mu^{\varepsilon} \delta H
\delta \varepsilon
                                       s^B \delta \tau
                                                        + g\delta B
                                       -g \delta \tau + v^{\tau} \delta B
\delta H
                                       c^B \delta \varepsilon
                                                       - h\delta B
\delta \tau
                                       -h \delta \varepsilon + v^{\varepsilon} \delta B
δН
                   =
```

In these relationships: (1) the shear strain components are defined by $\varepsilon_{ij} \equiv (1/2)(du_i/dx_j + du_j/dx_i)$, $i \neq j$, where the u_i 's are the components of the displacement vector, and (2) the shear stress components are not compressed (τ_{ij} may not equal τ_{ii} , allowing for body rotations).[1]

It is an accepted procedure to compress the $2^{\rm nd}$ rank strain and stress tensors to 6 dimension vectors and the $4^{\rm th}$ rank tensors into 36 component $2^{\rm nd}$ rank tensors. Employing the Voigt strain components defined by $S_{ij} \equiv du_i/dx_j + du_j/dx_i = 2\varepsilon_{ij}$ and the stress components given by $T_{ij} = \tau_{ij} + \tau_{ij}$, the first general elastomagnetic matrix listed above can be written:[2,3]

δS_{xx}		s_{II}^H	s_{12}^{H}	s_{13}^{H}	s_{14}^{H}	s_{15}^{H}	s_{16}^{H}	d_{II}	d_{21}	d_{31}	δT_{xx}
δS_{yy}		s_{12}^{H}	s_{22}^{H}	s_{23}^{H}	s_{24}^{H}	s_{25}^{H}	s_{26}^{H}	d_{12}	d_{22}	d_{32}	δT_{yy}
δS_{zz}		s_{13}^{H}	s_{23}^{H}	s_{33}^{H}	S ₃₄ ^H	s_{35}^{H}	s_{36}^{H}	d_{I3}	d_{23}	$d_{\it 33}$	δT_{zz}
δS_{yz}		s_{I4}^{H}	s_{24}^{H}	s_{34}^{H}	S_{44}^{H}	s_{45}^{H}	s_{46}^{H}	d_{14}	d_{24}	d_{34}	δT_{yz}
δS_{zx}	=	.S₁5	s_{25}^{H}	s_{35}^{H}	s_{45}^{H}	S ₅₅ ^H	s ₅₆ ^H	d_{I5}	d_{25}	d_{35}	δT_{zx}
δS_{xy}		s ₁₆ ^H	s_{26}^{H}	s ₃₆ ^H	s_{46}^{H}	s ₅₆ ^H	s ₆₆ ^H	d_{16}	d_{26}	d_{36}	δT_{xy}
δB_x		d_{11}	d_{12}	d_{I3}	d_{14}	d_{I5}	d_{16}	${\mu_{II}}^T$	${\mu_{l2}}^T$	${\mu_{l3}}^T$	$\delta\!H_{x}$
δB_y		d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	${\mu_{l2}}^T$	${\mu_{22}}^T$	μ_{23}^{T}	$\delta\! H_{ m y}$
$\delta \! B_z$		d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	μ_{I3}^{T}	μ_{23}^{T}	μ_{33}^{T}	$\delta\!H_z$

The remaining three elastomagnetic relationships follow in a similar manner, but they will not be considered here.

Note that in terms of the true tensor notation, this matrix becomes:[1]

$\delta arepsilon_{_{\! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! $		s_{II}^{H}	s_{12}^H	s_{13}^{H}	1/2S14 ^H	1/2S ₁₅ H	1/ _{2S16} H	d_{11}	d_{2I}	d_{3I}	$\delta au_{\!\scriptscriptstyle m xx}$
$\delta arepsilon_{yy}$		s_{12}^{H}	s_{22}^H	s_{23}^{H}	1/2S24 ^H	1/2S25 ^H	1/2S26 ^H	d_{12}	d_{22}	d_{32}	$\delta au_{\!\scriptscriptstyle {yy}}$
$\deltaarepsilon_{\scriptscriptstyle \! \!\! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! $		s_{13}^{H}	s_{23}^{H}	s_{33}^{H}	1/2S34 ^H	1/2S35 ^H	1/ _{2S36} ^H	d_{I3}	d_{23}	d_{33}	$\delta au_{\!\scriptscriptstyle zz}$
$\delta arepsilon_{y_Z}$		1/2S14 ^H	1/2S24 ^H	1/2S34 ^H	1/ _{4S44} H	1/4S45 ^H	1/ _{4S46} H	½d ₁₄	1/2d ₂₄	½d ₃₄	$\delta(\tau_{yz}+\tau_{zy})$
$\delta arepsilon_{ m zx}$	11	1/2S15 ^H	1/2S25 ^H	1/2S35 ^H	1/4S45 H	1/4S55 ^H	1/ _{4S56} ^H	½d ₁₅	½d ₂₅	½d ₃₅	$\delta(\tau_{yz} + \tau_{zy})$
$\delta arepsilon_{xy}$		1/2S16 ^H	1/2S26 ^H	1/2S36 ^H	1/4S46 ^H	1/4S ₅₆ H	1/4S ₆₆ ^H	½d ₁₆	½d ₂₆	½d ₃₆	$\delta(\tau_{zx} + \tau_{xz})$
δB_x		d_{II}	d_{12}	d_{13}	½d ₁₄	½d ₁₅	½d ₁₆	${\mu_{II}}^T$	${\mu_{l2}}^T$	μ_{l3}^{T}	$\delta\! H_{\scriptscriptstyle X}$
δB_{y}		d_{2I}	d_{22}	d_{23}	½d ₂₄	½d ₂₅	½d ₂₆	μ_{l2}^{T}	${\mu_{22}}^T$	μ_{23}^{T}	$\delta\! H_{\scriptscriptstyle m y}$
δB_z		d_{3I}	d_{32}	d_{33}	1/2d ₃₄	½d ₃₅	½d ₃₆	μ_{I3}^{T}	μ_{23}^{T}	μ_{33}^{T}	$\delta\! H_z$

This matrix has three components:

- 1. The 6 x 6 s submatrix is the compliance matrix; it has 21 independent components.
- 2. The 3 x 6 d submatrix is the piezomagnetic matrix; it has 18 independent components.
- 3. The 3 x 3 μ submatrix is the permeability matrix; it has 6 independent components.

IV. Effect of Symmetry

If we consider an isotropic material polarized with a magnetic field and uniaxial stress applied along the z axis, the matrix simplifies greatly. Recall that δS , δT , δB and δH are small quantities in the elastomagnetic formulation. If larger values of stress and field are used, as may be the case in high power sound projectors, this relationship yields average values that may be a function of stress and field amplitude. For this case, the first elastomagnetic matrix reduces to:

											_
δS_{xx}		s_{11}^H	s_{12}^H	s_{13}^{H}	0	0	0	0	0	d_{31}	δT_{xx}
δS_{yy}		s_{12}^{H}	s_{II}^H	s_{13}^H	0	0	0	0	0	d_{31}	δT_{yy}
δS _{zz}		s_{13}^{H}	s_{13}^{H}	s_{33}^H	0	0	0	0	0	d_{33}	δT_{zz}
δS_{yz}		0	0	0	S_{44}^{H}	0	0	0	d ₁₅	0	δT_{yz}
δS_{zx}	=	0	0	0	0 .	s_{44}^{H}	0	d_{15}	0	0	δT_{zx}
δS_{xy}		0	0	0	0	0	$s_{11}^{H} - s_{12}^{H}$	0	0	0	δT_{xy}
δB_x		0	0	0	0	d_{15}	0	${\mu_{l}}_{l}^{T}$	0	0	δH_x
δB_y		0	0	0	d_{15}	0	0	0	${\mu_{II}}^T$	0	$\delta\! H_{y}$
δB_z		d_{31}	d_{31}	d_{33}	0	0	0	. 0	0	μ_{33}^{T}	$\delta\!H_z$

Terfenol, on the other hand, is not isotropic, structurally or magnetically. Structurally the [112] direction is the preferred axis of growth with twinned platelets of $\mathrm{Tb}_{1-x}\mathrm{Dy}_x\mathrm{Fe}_2$ separated by eutectic sheets. In the large Terfenol samples, the platelets are not parallel throughout the sample and, to the lowest approximation, can be considered randomly oriented in the sample plane. Magnetically the magnetostriction constants are not isotropic, in fact, $\lambda_{111} >> \lambda_{100}$, which makes the piezomagnetic d constants very anisotropic. The approximation which most closely resembles the large diameter Terfenol-D rods in the transducer configuration consists of isotropy in the plane perpendicular to an applied field and stress H and T along the growth axis. (To date, it is impossible to prepare high quality large diameter [112], [111], or [110] single crystals.) Fortunately, for this case of magnetic field and stress applied along the [112] axis and assuming isotropy in the plane, the correct matrix is identical to the fully isotropic case shown above.[1] In the tensor notation, this can be rewritten:

$\delta arepsilon_{xx}$		s_{II}^H	s ₁₂ ^H	s_{13}^{H}	0	0	0	0	0	d_{31}	$d au_{\!\scriptscriptstyle { m XX}}$
$\delta arepsilon_{ m yy}$		s_{12}^{H}	s_{11}^H	s_{I3}^{H}	0	0	0	0	0	d_{31}	$d au_{\!\scriptscriptstyle { m yy}}$
\deltaarepsilon_{zz}		s_{13}^{H}	s_{13}^{H}	s_{33}^{H}	0	0	0	0	0	d_{33}	$d au_{zz}$
$\delta arepsilon_{yz}$		0	0	0	1/4S44 H	0	0	0	½d ₁₅	0	$\delta(\tau_{yz} + \tau_{zy})$
$\delta arepsilon_{zx}$	=	0	0	0	0	1/4S44 H	0	½d ₁₅	0	0	$\delta(\tau_{zx} + \tau_{xz})$
$\delta arepsilon_{ ext{xy}}$		0	0	0	0	0	$^{1}/_{4}(s_{11}^{H}-s_{12}^{H})$	0	0	0	$\delta(\tau_{xy}+\tau_{yx})$
δB_x		0	0	0	0	½d ₁₅	0	${\mu_{l}}_{l}^{T}$	0	0	δH_x
δB_y		0	0	0	½d ₁₅	0	0	0	${\mu_{l}}_{l}^{T}$	0	$\delta\! H_{y}$
δB_z		d_{31}	d_{31}	d_{33}	0	0	0	0	0	μ_{33}^{T}	$\delta\!H_z$

where we have set $d_{14} = -d_{25} = 0$, see Ref. 1.

V. Uniaxial Transduction Excitation

If in addition to the bias stress and field along the z axis, the a.c. fields and stresses are applied along the same (z) axis, only the following equations remain:

$$\delta S_{xx} = s_{13}^{H} \delta T_{zz} + d_{31} \delta H_{z}$$

$$\delta S_{yy} = s_{13}^{H} \delta T_{zz} + d_{31} \delta H_{z}$$

$$\delta S_{zz} = s_{33}^{H} \delta T_{zz} + d_{33} \delta H_{z}$$

$$\delta S_{yz} = 0$$

$$\delta S_{zx} = 0$$

$$\delta S_{xy} = 0$$

$$\delta B_{x} = 0$$

$$\delta B_{y} = 0$$

$$\delta B_{z} = d_{33} \delta T_{zz} + \mu_{33}^{T} \delta H_{z}$$

At this stage in the development of Terfenol, it is not possible to incorporate more terms and characterize the material better. Large diameter Terfenol-D, as prepared today and utilized in currently proposed high power sonar systems, can be fully characterized by the measurement of the following five coefficients:

- 1. $s_{33}^{H} = 1/E$, 2. $s_{13}^{H} = -v/E$, where v is Poisson's ratio,
- 3. d_{33}
- 4. d_{31} , 5. μ_{33}^{T} ,

and all five "constants" are functions of both H and T.

VI. Recommended Measurement Techniques

Assuming stress and field are applied only in the z direction and that the plane perpendicular to the stress is isotropic in the plane, Terfenol-D can be characterized in a straightforward manner: Strain gages are cemented both along the direction of applied stress and field and in the perpendicular direction. One large strain gage or a number of smaller gages can be used in order to obtain a good average of the constants. (Strain gages of course measure only the surface and it must be assumed that the material does not change radially.) One possibility is to apply a single strain gage around the circumference of the sample or average a number of smaller gages. An optical method or linear variable transformer method (LVDT) can also be used to obtain an average over the entire sample, at least in the z direction.

The applied field and stress can be measured by a number of conventional methods. One method is to employ a large face electromagnet whose pole faces are modified to apply compressive stresses of sufficient magnitude. A load cell measures the applied stress. *B* can be measured using a pickup coil wound around the sample; the field can be measured by a Hall probe. Alternatively, dual concentric coils can be would around the sample. By measuring the flux through both of the coils, both *B* and *H* can be calculated.

- 1. H is held constant and T is varied. The compliances s_{33}^H and s_{13}^H and Poisson's ratio v can be obtained from the parallel and perpendicular strain gages respectively. d_{33} can be obtained from this measurement if B is measured during the change in stress.
- 2. T is held constant and H is varied. The piezomagnetic constants d_{33} and d_{13} , are obtained from the parallel and perpendicular strain gages respectively. The permeability μ_{33}^{T} is obtained by measuring B.

These measurement techniques are also valid for the lower hysteresis Tb_xDy_yHo_zFe₂ quaternary samples that are prepared using the same processes as the Terfenol-D samples.

VII. Numerical Examples

As discussed earlier, the "constants" in the elastomagnetic matrices are not really constants. Fig. 1 shows the variation of the d constant and μ relative (= μ / μ_0) as a function of H for a large diameter Terfenol-D rod. The d "constant" varies by almost a factor of 10 as H varies and μ relative varies by a factor of 2.5. Fig. 2 shows the compliance s_{33}^H as a function of T at a field of 147 kA/m (~1850 Oe). s_{33}^H varies by a factor of 1.8. For these numerical examples we must pick a field and a stress and find the appropriate parameters. For our field H, we choose the maximum d_{33} point, 151 kA/m (~1900 Oe), d_{33} = 9.1 x 10^{-9} m/A. The relative μ_{33} at the same point is 3.8. For a compressive stress of 56 MPa, s_{33}^H is 3.2 x 10^{-11} m²/N. We

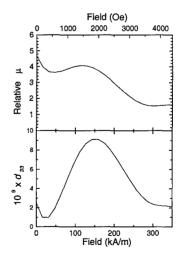


Fig. 1. Relative μ and d vs field at 56 MPa.

use the values of Poisson's ratio v = 0.45 and the value of d_{33} / $d_{31} = -2.0$ given by Claeyssen[4]. This gives:

1.
$$s_{33}^H = 3.2 \times 10^{-11} \text{ m}^2/\text{N},$$

2.
$$s_{13}^{H} = -1.44 \times 10^{-11} \text{ m}^2/\text{N}$$

3.
$$d_{33} = 9.1 \times 10^{-9} \text{ m/A},$$

4.
$$d_{31} = -4.6 \times 10^{-9} \text{ m/A}$$
, and

5.
$$\mu_{33}^T = 3.8 \ \mu_0 = 4.8 \ \text{x} \ 10^{-6} \ \text{T-m/A}.$$

Assuming that $\delta H_z = 36$ kA/m (~450 Oe) and $\delta T = 0$, we have:

$$\delta S_{xx} = \delta S_{yy} = d_{31} \delta H_7 = -6.91 \text{ x } 10^{-5} \text{ m},$$

$$\delta S_{zz} = d_{33} \delta H_z = 3.26 \text{ x } 10^{-4} \text{ m}, \text{ and}$$

$$\delta B_z = \mu_{33}^T \delta H_z = 0.172 \text{ T}$$

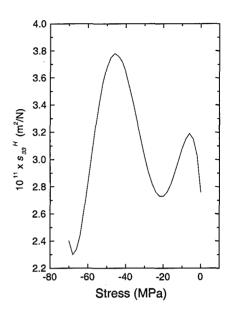


Fig. 2. Compliance vs. stress for a large, Bridgman, Terfenol-D sample at 147 kA/m.

Obviously, if there is no magnetoelastic coupling, which corresponds to all of the d's being zero, all of the δS 's are also zero.

VIII. Summary

Large diameter Terfenol-D samples with the stress and field applied in the z direction and isotropic properties in the x-y plane can be described by only five parameters. These parameters can be measured with a pickup coil and strain gages mounted parallel and perpendicular to the z axis.

If the preparation methods improve in the future such that high power transducer material can be made similar to the FSZM materials or [111] and [110] single crystals can be produced, then the symmetry in the x-y plane is broken and a larger number of constants must be used to characterize the materials.

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